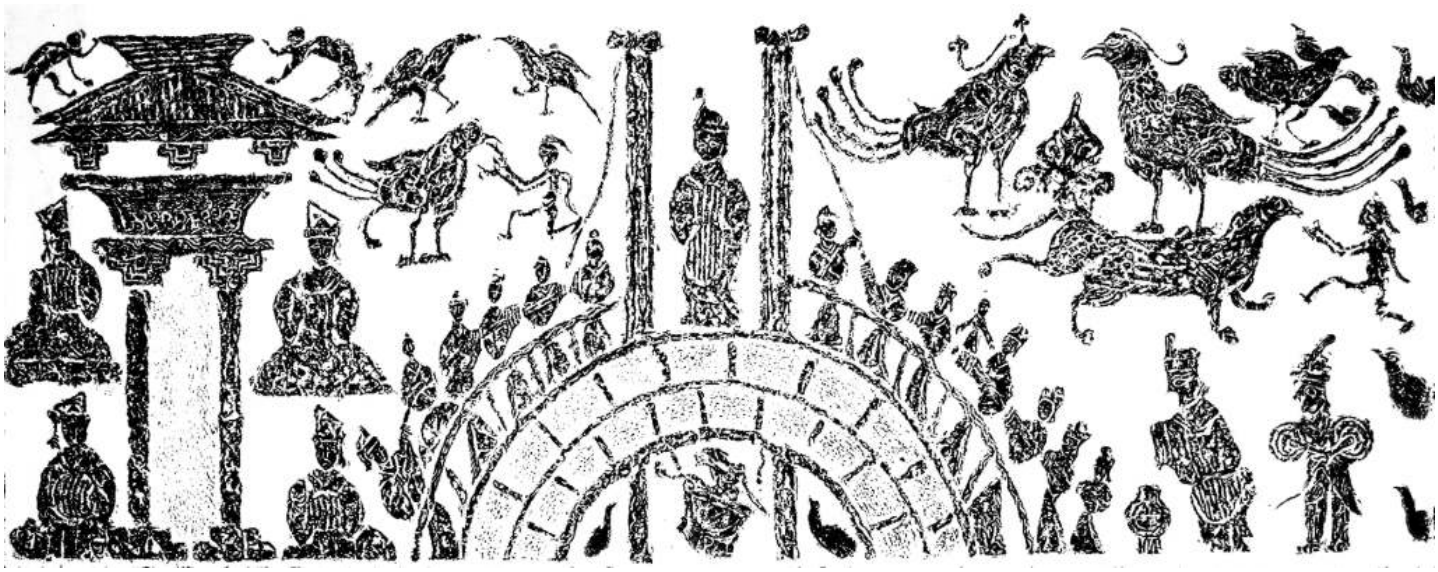
Tripod Notation

A Magic Cauldron of Musical Possibilities

Graham Breed



Originally the tripod was an ancient cooking utensil and was often used to contain offerings. Later it became a symbol of power and reign. Throughout the three dynasties of Xia, Shang and Zhou, there were nine tripods taken as a symbol of the imperial power. Whoever owned them would be the one to seize the throne.

Travel Round Jiangsu

Contents

1 Preliminaries

千里之行始於足下[1]

Chinese proverb

1.1 Steps and Leaps

CONTRAPUNTALISTS of old made a distinction between "steps" and "leaps". Steps are the building blocks of melody and leaps are the building blocks of harmony. With conventional notations, a step moves you from one scale step to the next and a leap misses out at least one scale step. Hence the distinction between steps and leaps is built into the notation.

Tripod notations are based on the idea that an octave is divided into three *feet*. A leaps takes you from one foot to another and a step takes you to another note on the same foot.

A consequence of this is that the ninth degree of a chord is not on the same foot as a tonic. By the melodic definition, an octave to a ninth is a second and so a step. By this wacky definition, the seventh is on the same foot as the tonic and the ninth is on the next foot, the same as the third. That may sound strange but it's the way the notation works.

The fifth degree of a chord is on the foot below the octave. So, a single leap takes us from the fifth to the octave. This interval is a perfect fourth. It follows that a perfect fourth is a single leap, giving us a concept of "single leap" that is distinct from that of "third". If a triad is built of two consecutive, single leaps it follows that a suspended fourth chord is a kind of triad.

Of course, an interval between a third and a fourth is a second. If the third and fourth are on the same foot they must be separated by a step. So the "seconds" of conventional notations may be either "steps" or "leaps" in tripod notations. This distinction means that tripod notations carry different information to that of conventional notations.

1.2 The Lattice

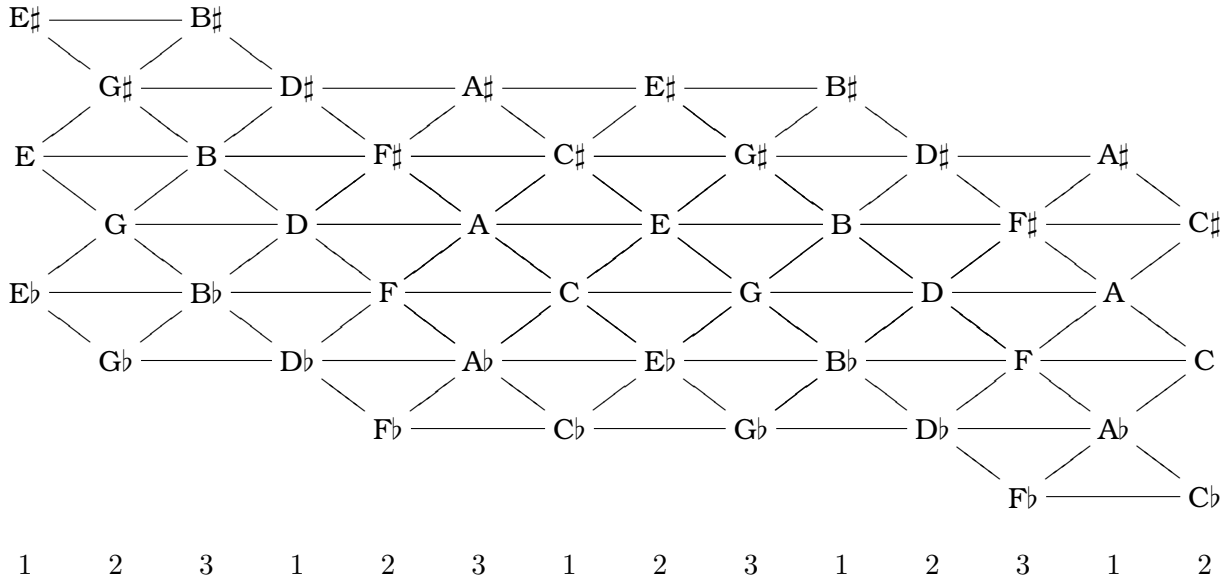
YOU CAN SEE a simple *lattice* or *Tonnetz* in Figure 1.1. This is a simple structure that may cause an undue amount of excitement. It's made up of triangles that represent major and minor triads. Each basic consonance – perfect fifths and major and minor thirds – runs in a different direction. You can use it to follow traditional chord progressions.

Where exact tuning is the issue it's important remember that, in general, two notes with the same name that sit on different parts of the lattice are not really the same pitch. The exception, where they are the same, is what I call *meantone temperament*. Most of this exposition is not about meantone temperament so the lattice comes first and the traditional note names are only convenient tags.

To help distinguish notes with the same name, there are indications at the bottom of which foot the note is on. Whenever you see a pair of notes in Figure 1.1 that seem to be the same, each falls on a different foot. Because of this you can use the traditional note name along with the foot indication to

[1] A Journey of five hundred kilometres begins with a step.

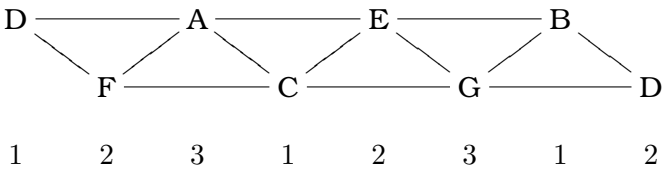
Figure 1.1: A section of the lattice with meantone names



uniquely identify a pitch. Eventually you will still find duplicates but you get a lot further than you would with only the meantone names.

The full lattice would run off the page at the left and right edges (transposition by fourths and fifths). To go further up and down (transposition by thirds and sixths) you need to use double sharps and flats. I left them off the diagram to make it simpler.

In the following lattice you can see that the key of C major requires two different tunings of the note D for each chord to follow its theoretical tuning. However, each of these notes is on a different foot. If you want a D minor chord, you always take D from foot 1. And if you want a G major chord, you always take D from foot 2. Harmony may take you outside this set of notes. When that happens, you need to re-define the feet (which foot you start with is arbitrary).



1.3 Nine-Limit Harmony

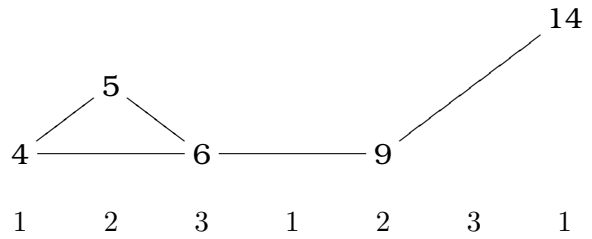
THERE ARE MANY reasons for exploring microtonality. Tripod notation is primarily concerned with the search for new harmonies. Specifically, it's intended for *nine-limit harmony*. I'll call well tuned, traditional major and minor chords *Didymic*. These are the simple triangles that make up the lattice.

The numbers here refer to harmonics. The root, or fundamental, of a tone is one. Harmonic number two is an octave above it, and harmonic number three is a fifth higher than harmonic number two. Harmonic number four is two octaves above the fundamental. Multiplying the harmonic by two always raises the pitch by an octave. Because of this, the interesting new pitches correspond to odd numbered harmonics. The harmonic limit[2] is the largest odd number you need to define an *otonality*[3] by harmonics.

The otonality that defines Didymic (or *five-limit*) harmony is a major triad. The root has a harmonic number of four (two octaves

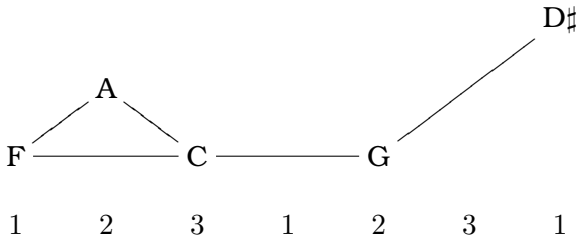
[2] As Harry Partch called it; see *Genesis of a Music*
[3] *ibid*

above the theoretical fundamental). The fifth has a harmonic number of six (an octave above three). The third has a harmonic number of five. Different inversions give different harmonic numbers but the five-limit is preserved. A minor chord has the same intervals as a major chord but in a different order, so it still belongs to the five limit.



This diagram shows a nine-limit otonality on the lattice. The first three notes—four, five, and six–describe a major triad. The nine is a fifth above the six or a whole tone and an octave above the root. Then we have a last note, with a harmonic number of fourteen, which is twice seven. Seven is an odd number larger than five, so this pitch is new.

The lattice is defined for Didymic harmony, so the fourteen doesn't really fit on it. It's placed according to *marvel temperament*. You can think of this as setting the interval between harmonics nine and fourteen as being two Didymic major thirds. That means the interval between the harmonic of number seven (an octave below the fourteen) and the harmonic of number nine is a kind of major third. Because the true Didymic major third is slightly smaller than a third of an octave, this new nine-limit major third is slightly larger to balance it out. Hence it's sometimes called a *supramajor third*. Marvel temperament allows two major thirds and a supramajor third to add up to an octave.



If all this talk of harmonic numbers is too much for you, take a look at the diagram above. Here, a nine-limit otonality is written as an extension of an F major chord. The seventh degree, or harmonic number fourteen, is best written as D sharp. Strictly this is an augmented sixth rather than a seventh, but it happens to be a better fit in meantone temperament (which, you may recall, works with conventional note names). A minor seventh is a little to large. The foot indications show that the D sharp is on the same foot as the F, so in a sense they are a seventh apart.

1.4 Magic Temperament

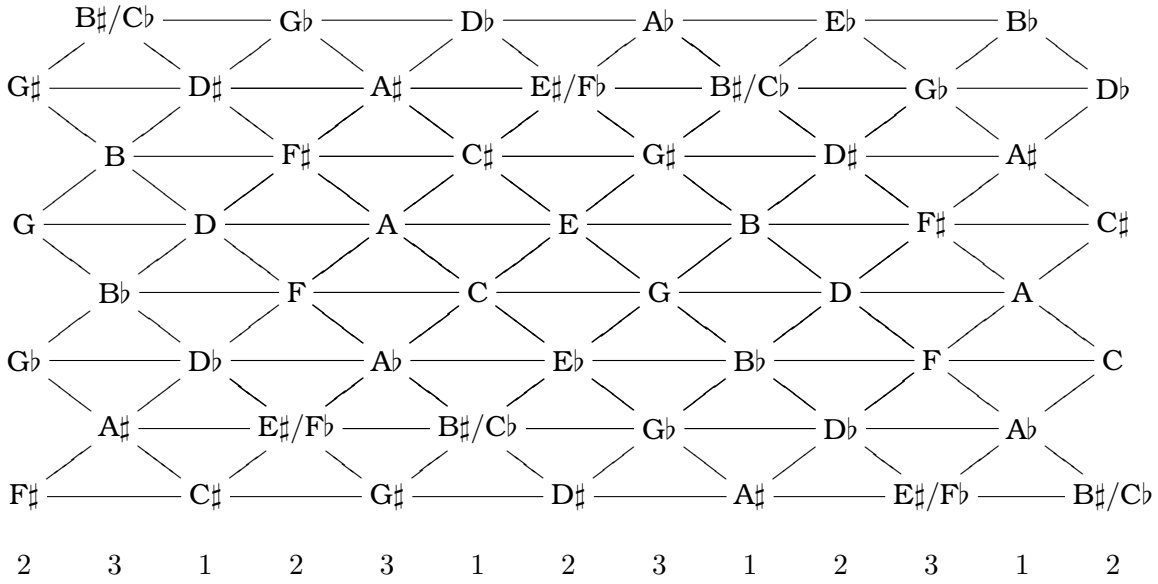
ALL THIS TALK of meantone temperament and "conventional note names" may have given you a warning that we're about to step outside this bubble of conventionality and enter a new class of temperaments. That is indeed the case, and the new temperament class is called *magic*.

Magic and meantone both work with scales of nineteen notes. As with nineteen note equal temperament, E sharp and F flat end up as the same pitch. So, whereas twelve note equal temperament divides the whole tone into two equal parts, nineteen note equal temperament splits the semitone in half instead. The other diatonic semitone, between B and C, is split the same way, to make B sharp and C flat the same pitch.

The term "diatonic semitone" is itself part of the meantone world. I call the corresponding interval in magic temperament a *toe*. The toe can be split into two equal parts in any magic temperament, not only nineteen tone equal temperament. Hence a toe consists of two equal *semitoes*.

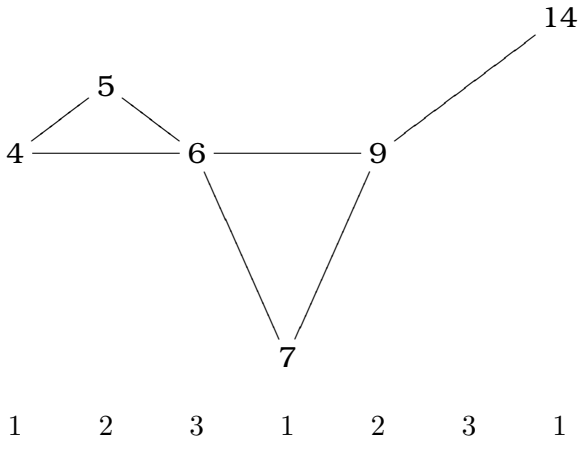
Figure 1.2 uses the nineteen note magic scale to place meantone names on a magic lattice. Sometimes, two different magic pitches will have the same meantone name. However, if the meantone names are the same and the two pitches are on the same foot, then they really are the same pitch in magic temperament. (Or, at least, the two pitches are separated by a whole number of octaves, if you interpret the lattice as distinguishing

Figure 1.2: A section of the lattice with magic equivalences



pitches an octave apart.)

These equivalences of magic temperament mean that the nine-limit otonality can be written in a different way to plain mavel temperament, and this is shown below. The marvel equivalence still applies, so the fourteen hasn't moved, but there's an additional note with a harmonic number of seven in the diagram.



Ignoring the fourteen, then, this chord is made up of two triads. Firstly, the usual major triad includes the pitches with harmonic numbers of four, five and six. The top half of the chord—harmonics six, seven and nine—form a kind of minor triad. As the interval between harmonics seven and nine is a supramajor third, the interval between harmonics six and seven must be smaller than a minor third to provide balance, and so is a *subminor third*. This chord, formed by placing a perfect fifth and a subminor third above the root, is then a *subminor triad*. The nine-limit otonality is formed by a major triad on the root and a subminor triad on the fifth. In both cases, the three notes of the triad are on different feet.

If you find the major and subminor triads on the full lattice, you should see that the subminor triad is formed by lowering the third of a minor triad by the same amount that a minor third is smaller than a major third. The difference is the interval I called a semitoe before. So we can say that a minor third is a semitoe smaller than a major third, a subminor third is a semitoe smaller than a minor third, and a supramajor third is a semitoe larger than a major third.

Raising a supramajor third by another semitoe gives a perfect fourth. To see this, find the F sharp at the bottom left-hand corner of Figure 1.2. The foot indications at the bottom show that it's on foot 2. Directly to its right is a C sharp on foot 1. So, F sharp on foot 2 is a perfect fourth above C sharp on foot 1. The column to the right of the C sharp, which contains the notes a single leap above foot 1, is also foot 2. Follow that column up to see that the major third above C sharp is

E sharp, which is identical to F flat on this foot. The supramajor third above C sharp is equivalent to F. And the semitoe above that is F sharp, which we already saw is the perfect fourth above C sharp when it's on foot 2.

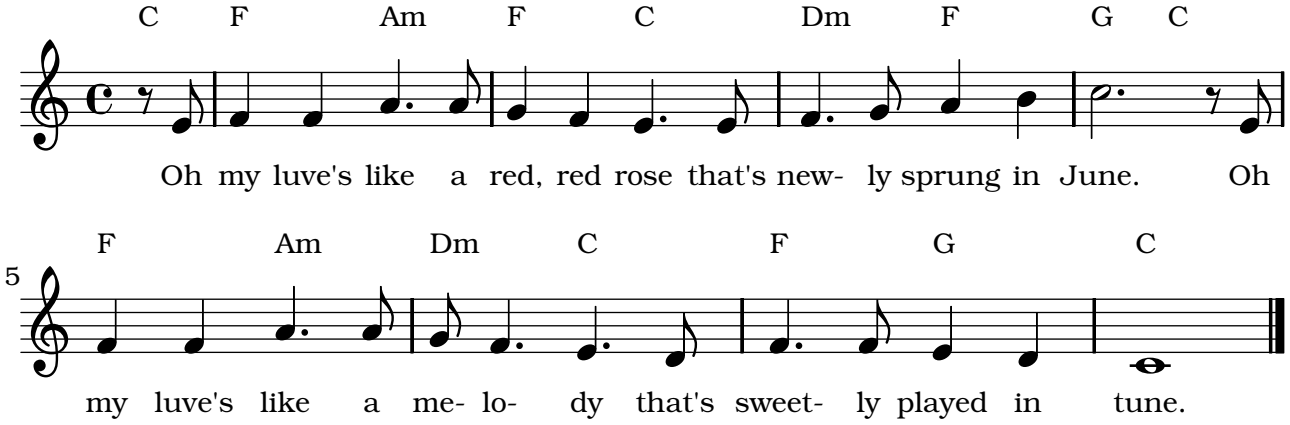
Symmetry then tells us that the semitoe below the subminor third must be the whole tone that you find between a pure, perfect fourth and fifth. Magic temperament has two different sizes of whole tone in the nine-limit that both end up as major seconds in meantone. This is the larger one, which is called a *major tone*. In magic temperament, a major tone is a toe smaller than a minor third.

The smaller whole tone, or *minor tone*, is a step; that is, both notes are on the same foot. On the left hand side of the lattice, you can see that the interval between F sharp an G sharp when on the same foot is three semitoes. Hence the minor tone in magic temperament is three semitoes.

1.5 A Red, Red Rose

As this exposition is about new ways of writing music, it makes sense to have some music to write. I'll use *A Red, Red Rose*, a song collected by Robert Burns, as a common example. You can see the vocal line and chords as I like them[4] below. There are more verses and they follow much the same pattern.

The chords are all diatonic and, as we'll see later, all work together in a general Didymic system. That means it's still easy to read in tripod notations. Nine-limit harmony will tend to have more inherent complexity, and that complexity adds to the complexity of the notation. So this example is a little simplistic. But, still, it's real music.



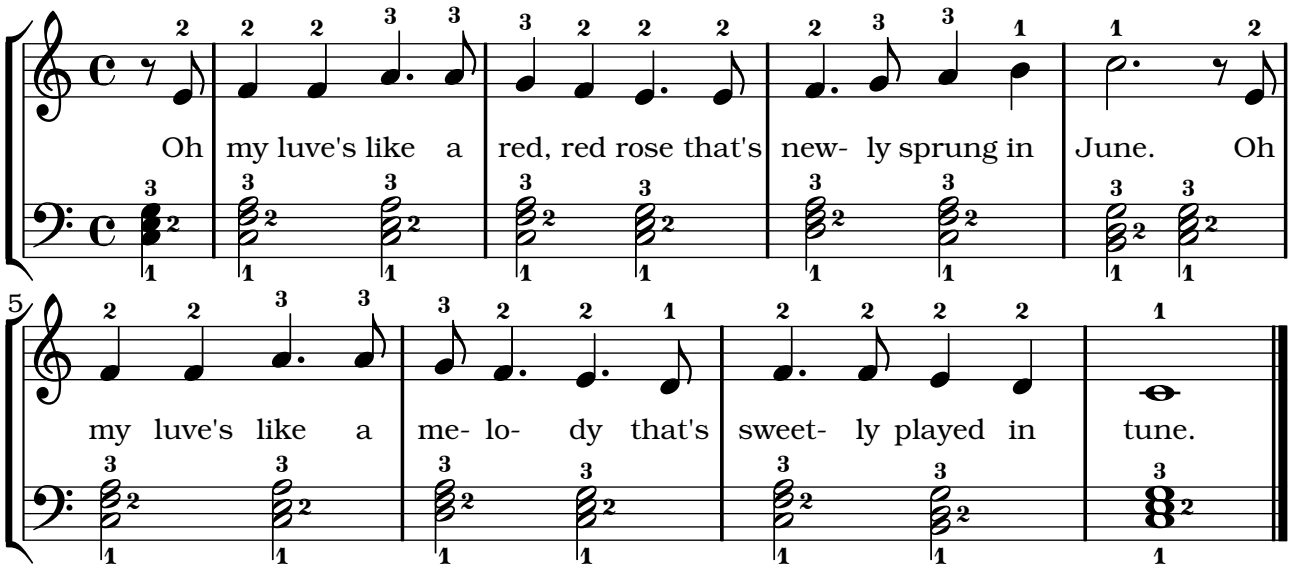
[4] Maybe this isn't the tune that's commonly used for the song. I don't care, and in fact I deliberately avoided looking at the song in any form of notation so that I can't be accused of copying a particular arrangement. So there.

2 Tricycle Notation

Now you may try to subtract it
But it just won't go away
Three times one? (What is it? One, two, three!)
And that's the magic number

De La Soul, The Magic Number

Figure 2.1: *A Red, Red Rose* with tricycle "fingerings"



BEFORE DEALING WITH the true tripod notation, I'll take a diversion and introduce a compromise that keeps the traditional note names and adds a three-fold division of the octave. The result is that you have, conceptually, three different meantone scales. Things like this are sometimes called "bike chains" which leads to the name of the notation system: *tricycle notation*.

If we're looking at a tricycle rather than a tripod, the three things we divide the octave into must be wheels rather than feet. That's only a difference in naming. The tripod's feet are really the same as the tricycle's wheels and you could even call tricycle notation a kind of tripod notation.

The structure of tricycle notation follows directly from the lattice as in Figure 1.1. Notes in the same column are in the same wheel, and the annotations at the bottom tell you which wheel is which. All we need to do is write the music as normal and add the same annotations to the score as are on the lattice. The easiest way to do this with standard software is to treat the wheel indications as fingerings.[1] Even with a crazily extended metaphor the wheels (or feet) are not fingers, so I'll call the annotations "fingerings" with the scare quotes always present.

You can see *A Red, Red Rose* marked up this way in Figure 2.1. The pattern of the chords is very simple, because the notes are always on wheels 1, 2 and 3 counting up. That's because of the conservative way

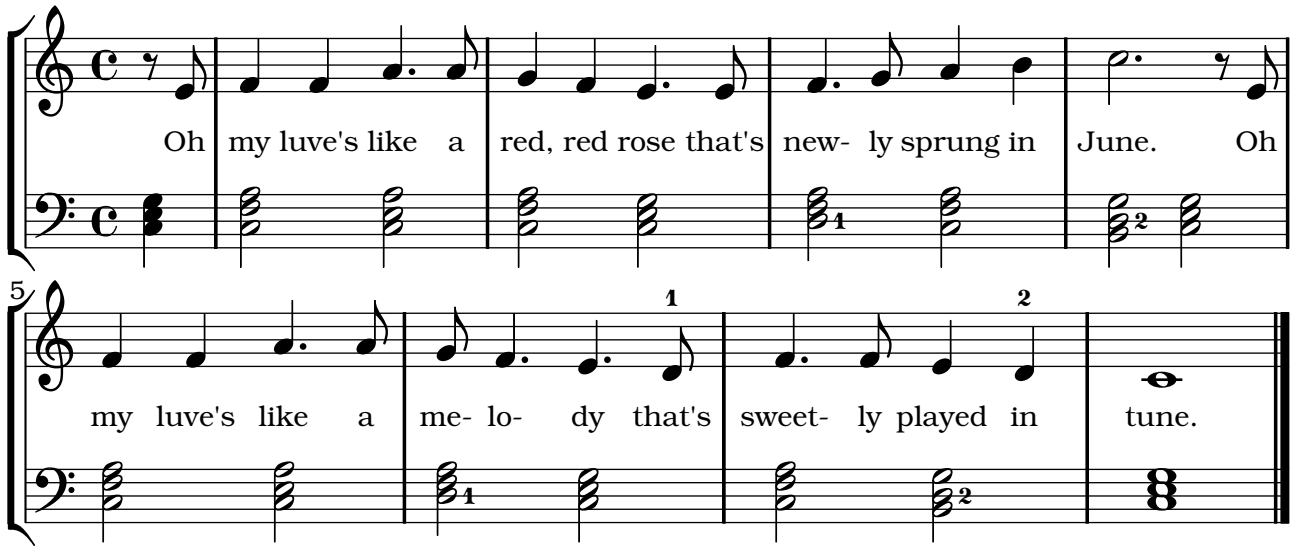
[1] I'm not sure it's the best way. Maybe one day I'll explain some variants. I think using "fingerings" is easy to understand as well as implement, though, so it works well enough in this context.

I voiced them. Think of the chords as three individual voices, and each moves by steps (in the tripod sense) without crossing. So each chord-voice stays on the same wheel. The vocal line covers a wider range and the wheels are mostly determined by the chords.

The "fingerings" add a lot of clutter, so in Figure 2.2 I only included them for ambiguous notes. Which is to say only for D because D is the ambiguous note in a C major scale. Every other note always sounds on the same wheel. As that's the wheel you'd expect for C major there's no need to keep reminding the reader of it.

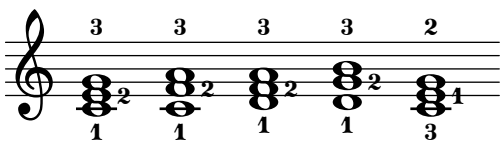
Note that the D of "that's" (bar seven) is truly ambiguous. There isn't a D in the chord it sounds over to fix it to either wheel. I've set it to be a major sixth above the F.

Figure 2.2: *A Red, Red Rose* with tricycle "fingerings" for ambiguous notes



2.1 Pitch Drift

THERE ARE SOME chord sequences that work in meantone temperament but don't end up on the same note as they started in a general Didymic tuning. They're called *comma pumps* because they cause the pitch to rise or fall by a small interval or *comma*. The popular I IV ii V I comma pump is shown below. In tricycle terms, it begins with C on wheel 1 and ends with it on wheel 3. The drift is unavoidable if you common notes in adjacent chords stay on the same wheel (which means they keep the same pitch).

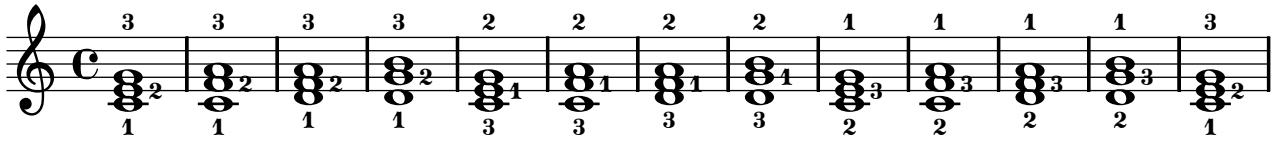


The same comma pump, repeated three times, is shown later. The notes at the end should have drifted by three commas. In fact, they end up on the same wheels that they started on. This shows a strange feature of tricycle notation, and another reason for its name: its cyclical. Although notes that differ by a single comma are distinguished, pitches three commas apart are written identically.

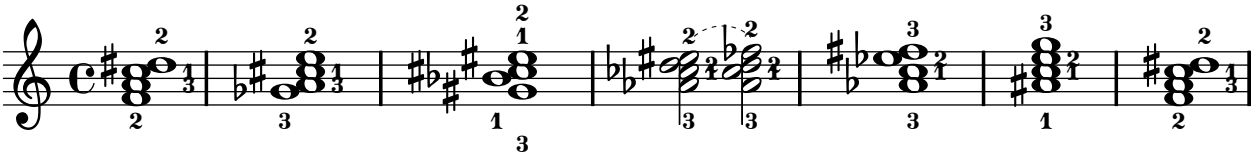
If you followed the example through and worked out the pitch of each note in a static tuning system that preserves the commas, you'd see that the pitch drift is still unavoidable. The tricycle notation determines the pitch of each note relative to those around it. It's only when you compare notes from different contexts that their pitches relative to each other become ambiguous.

This hiding of cumulative pitch drift can be a problem if you want a strict, static tuning. But it also has its advantages. If you

want the musicians to preserve the Didymic or marvel tuning of each chord as well as they can but to remove pitch drift, the notation matches what they should be doing. Unlike most extended notations, where an accidental symbol is added to show comma shifts relative to a fixed scale, the notation doesn't get more complex the further the theoretical pitch drifts from its starting point.



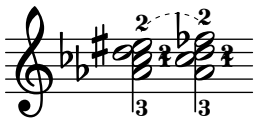
2.2 The Magic Tricycle



THE EXAMPLE above shows a magic comma pump in tricycle notation. In a general marvel tuning, it wouldn't end up where it started if the pitches of common notes in adjacent chords were preserved. In any magic tuning it works without pitch drift.

The root of each chord other than the last is a Didymic major third above the one before. Each chord is the same type—a Didymic major triad with an added harmonic seventh (written as an augmented sixth). That makes it easy to follow, and it gives an idea of what nine-limit harmony will look like in the notation. The last two chords are a perfect fifth apart, so this is an authentic cadence.

Writing it in meantone requires approximations from nineteen note equal temperament. That's made explicit in the bar with two dash-tied chords, like this:



The first chord has an E sharp and the second an F flat. Both have the same pitch on a nineteen note scale and so, with the "fingerings", are the same pitch in magic temperament. The dashed slur line is to indicate an approximation, and so a tie in magic temperament.

3 The Tripod Staff

> I've devised a system to totally revolutionize music . . . I've decimalized it. Instead of the octave, it's the decatave. And I've invented two new notes: H and J.
>
> *Holly, Red Dwarf II:1 (Grant & Naylor)*

TRIPOD NOTATION is written on a three line staff. Each line corresponds to a different foot. That means that the whole staff covers an octave. You can write a note on the line, above the line, or below the line. All three pitches are on the same foot.

The pitches of lines within a staff are separated by a major third. Between one staff and another—that is, between registers—is a river. The river adds an extra semitoe to notes that cross it. That means that adjacent lines on different staves are separated by a supramajor third.

A note on the line and a note in an adjacent space are a toe apart. You can write the semitoe between them using accidentals. That gives you a string of semitoes on each foot. Think of it as a tripod with three feet, each of which has three toes. You can write notes on or between the toes.

Because there's a space both above and below each line, there are two spaces between pairs of lines. A note written at the top of this space belongs to the foot above, and at the bottom of the space it belongs to the foot below. The pitches in the same space are separated by an interval slightly larger than a semitoe.

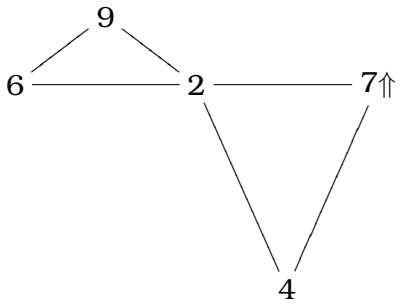
This summary assumes magic temperament, and tripod notation is primarily designed to work with magic, but it can still be used for music in any Didymic or marvel tuning.

3.1 Tripod Pitches

THREE FEET TO an octave and three notes on each foot gives you nine notes within the octave without accidentals. I give them numbers from one to nine in order of ascending pitch, with the river between notes nine and one as you ascend. I chose them so that the pitches are roughly evenly spaced which helps to preserve contours. You can call them *the tripod nominals* or *the tripod scale*.

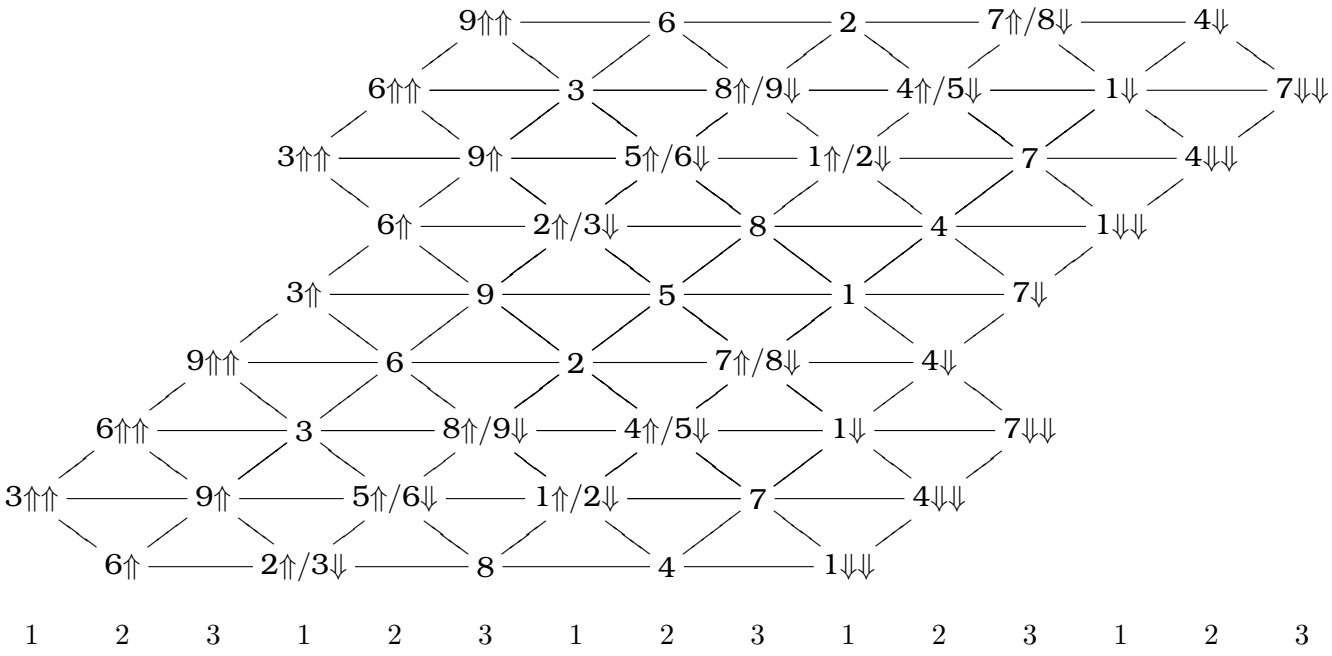
To see how nine-limit harmony comes out with these nominals, look at the diagram below. The ninth degree of the chord is a semitone above the seventh tripod nominal, so I used a double shafted arrow to raise the pitch accordingly. The same arrows are in Figure 3.1 along with arrows pointing down to lower a pitch by a semitoe.

Because a toe equally divides a semitoe, and adjacent scale degrees are often a toe apart, there are some pitches that can be written on two different scale degrees. I show both alternatives on the lattice. Having these choices means you can adjust the spelling of a chord to preserve the pitch contour and help avoid collisions.



The foot indications at the bottom are redundant now. Notes one to three are always on foot 1 and so on. They may help you to

Figure 3.1: A section of the lattice with tripod scale degrees



reconcile the tripod pitches with the meantone pitches in Figure 1.1. I've matched up tripod and tricycle notations so that C is on the first line, and so equal to tripod note two.

There are simple rules that tell you how to write an interval with tripod pitches. For example, a major third is three scale steps and leaps between adjacent feet. A minor third is a semitoe smaller than the major third. Where an interval crosses the river, you adjust by removing a semitoe. Hence the major third above note nine is a semitoe below note three.

The lattice continues above and below the section shown in the example. However, the pitches really do run out on either side, if you only use single and double semitoe shifts. This reflects the fact that perfect fifths are not so simple an interval in magic temperament as they are in meantone. Music that uses long chains of fifths is not likely to work well in tripod notation, but music that uses long chains of major thirds may work better than in standard, meantone-based notation.

I only showed full toe shifts where they describe a pitch that can't be written any other way. All of the tripod nominals can be written in terms of a double semitoe shift from at least one other scale degree. You aren't likely to use those equivalences very often and it would clutter up the lattice to show them all.

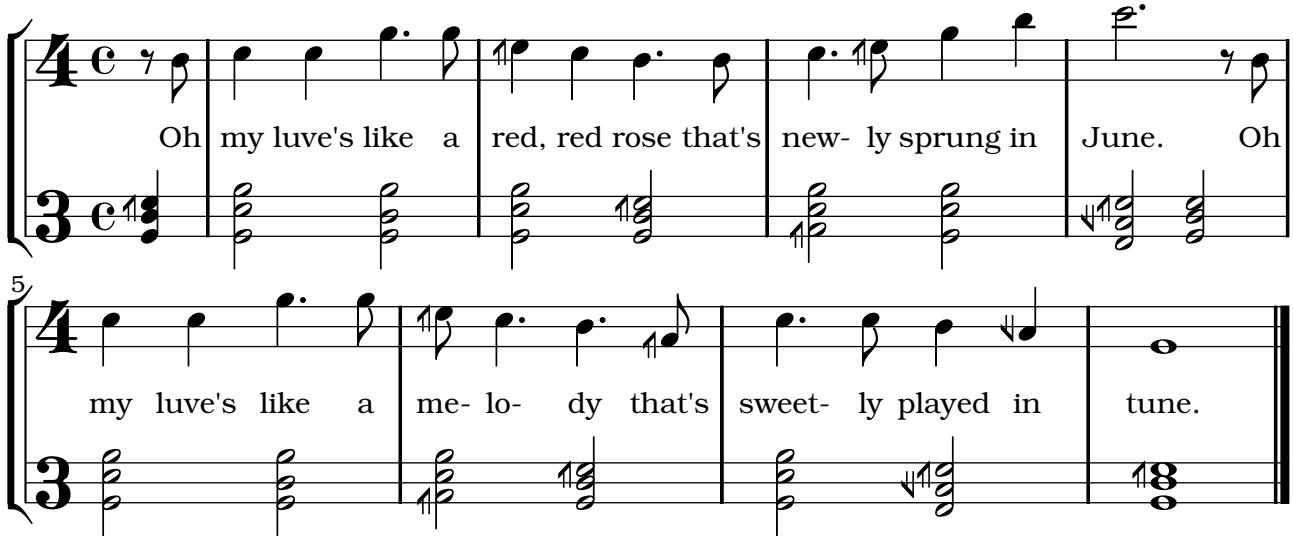
3.2 Semitoe Accidentals

I CHOSE MY accidentals from the Sagittal system of Dave Keenan and George Secor.[1] A magic semitoe lies between a usual semitone and quartertone, and so could be either a single or double shafted arrow in Sagittal. I chose one of the smallest and most important double shafted arrows.

You can see *A Red, Red Rose* in tripod notation with Sagittal accidentals in Figure 3.2. The big numerals show the register. Clef four is an octave above clef three and you can generalize this in an obvious way. Clef four corresponds to the bottom of the usual treble clef. There's a ledger line that no note can be placed on to mark the river.

[1] http://sagittal.org/

Figure 3.2: *A Red, Red Rose* in magic tripod notation



3.3 Magic Scales

YOU CAN SEE some important magic scales in Figure 3.3 in both ascending and descending forms. First off, the Huating scale has seven notes, and is in that sense comparable to the traditional diatonic. But its pitches are spread unevenly. Its step sizes are either a semitoe (smaller than a semitone) or a minor third.

The nineteen notes of Pengcheng are almost evenly spaced. It's too big to function as a mode but good as a set of notes to tune an instrument to. If you write it in tricycle notation with magic equivalences, each conventional name always refers to a pitch on the same foot. In tripod notation, it can be written with all the tripod nominals, all pitches a single semitone either higher or lower than a nominal, and one additional note between the last and first tripod nominals.

The table below shows the melodic structure of Pengcheng. An "s" is a step of a semitoe and an "S" is slightly larger. You can see that each "S" takes you from one foot to another. Figure 3.3 shows the two forms of Pengcheng in tripod notation, with upward shifts used in the ascending scale and downward shifts in the descending scale.

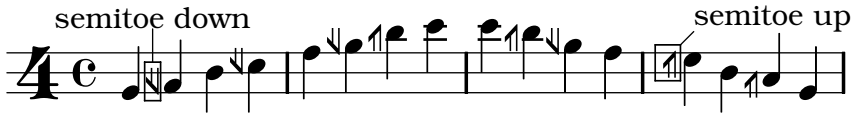
Haizhou has three more notes (one on each foot) and includes pairs of pitches that would be approximated the same way in meantone. That means you can use it to write diatonic music where a conventional note name is associated with more than one pitch. It always requires at least one full toe shift, to fill in the gap between the lowest and highest tripod nominals. It can be written with all the nominals and all possible single-semitoe shifts along with this single full toe shift. Figure 3.3 shows the two ways this can be done, with a pitch a full toe from either the first or last tripod nominal, for the ascending and descending scales. *A Red, Red Rose* is a subset of Haizhou in Figure 3.2, but doesn't require the full toe shift.

1	1⇑	2	2⇑	3	3⇑	4	4⇑	5	5⇑	6
	s	s	s	s	s	S	s	s	s	s

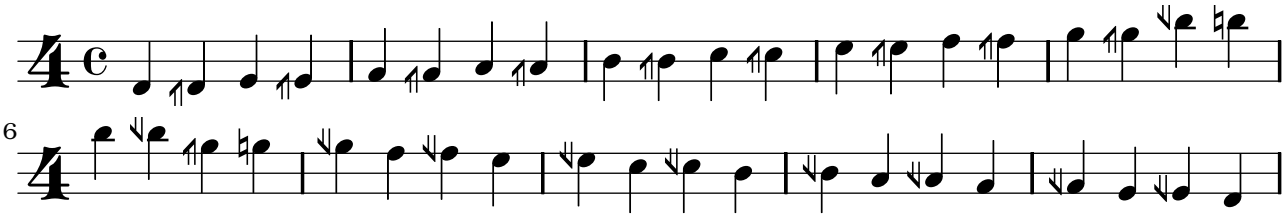
6	6⇑	7	7⇑	8	8⇑	9	9⇑	1⇓	1
	s	S	s	s	s	s	s	S	s

Figure 3.3: Some magic scales in tripod notation

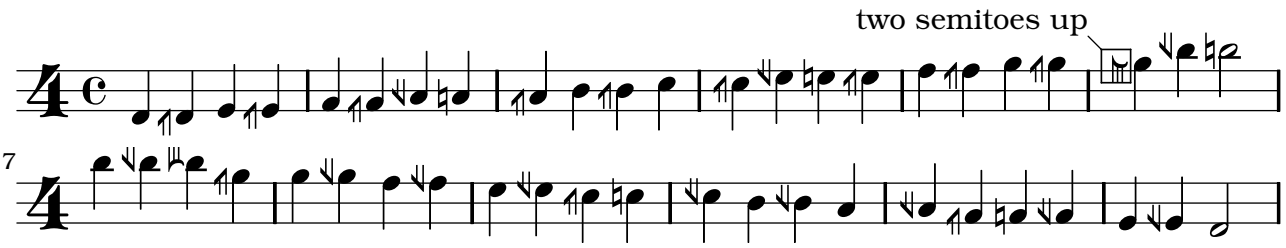
“Huating” scales of seven notes



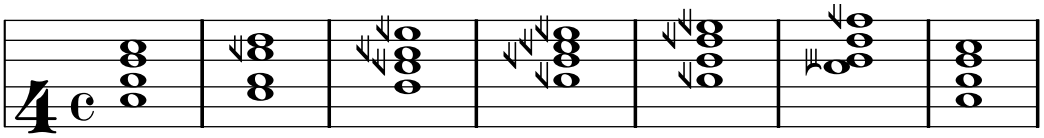
“Pengcheng” scales of nineteen notes



“Haizhou” scales of twenty-two notes



3.4 Crossing the River



When a part doesn't sit squarely on either side of the river, you can extend that staff to cover two octaves. That's what I did in the example above. It shows the same magic comma pump as in Section 2.2. Extra lines are added for notes above clef four, and there's no need for a ledger line to mark the river that's now in the middle of the staff. You can see where the river is because the lines are more widely spaced on either side of it.

As we've got two octaves to play with now, why not duplicate notes in octaves to put the roots in the bass?

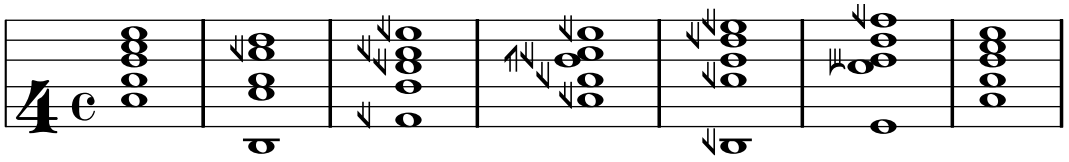
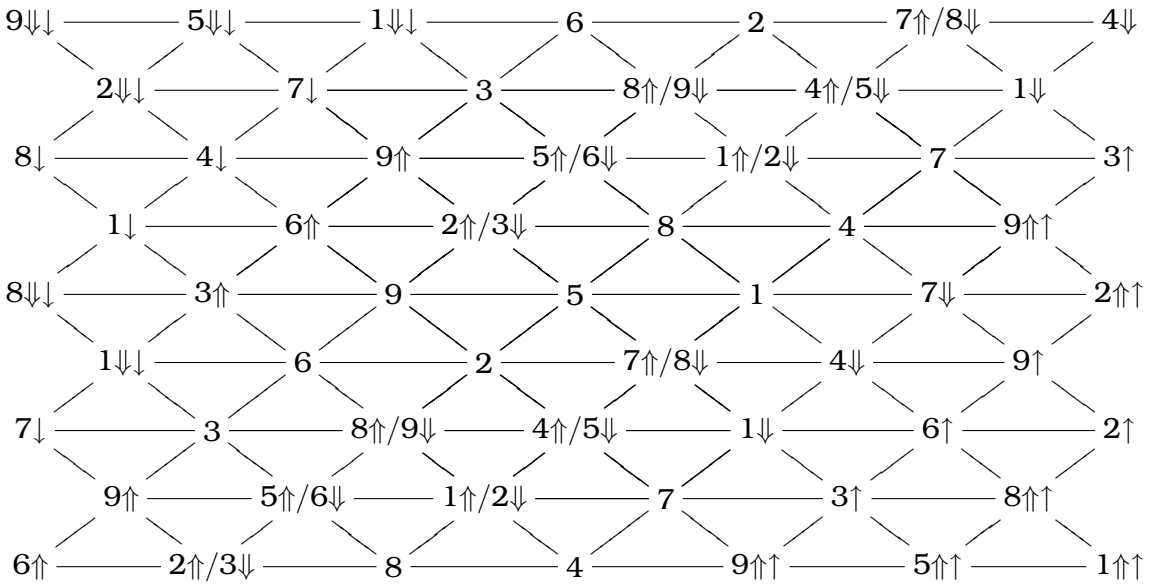


Figure 3.4: A section of the lattice with inch shifts

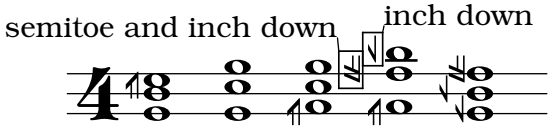


3.5 Inch Shifts

THE INTERVAL between two notes written at different positions in the same space is slightly larger than a toe. I call the amount by which it exceeds a toe an *inch*.

You can describe more pitches by writing shifts of an inch, as in Figure 3.4. The single-shaft arrows show raising and lowering by an inch. A lot more pitches can be written this way than with only single and double semitoe shifts. You still have to combine inches and semitoes to cover the Haizhou scale in Figure 3.3 though.[2]

Here's a Didymic comma pump written with inch shifts.

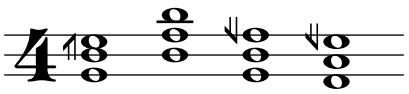


There's another new symbol for a semitoe and and inch combining in the same direction. You may end up using it more often than the single inch shifts because it allows you to reach notes that lie on the edges of feet, which are the most likely notes to require re-spelling. They also help you write notes on either side of the river in the correct pitch order.

3.6 Marvel Spellings

SO FAR I've described the pitches in terms of magic temperament, where each toe can be divided into two equal semitoes. Like tricycle notation, the tripod staff can be used to specify pitches in any marvel temperament. This can work because there are two ways of spelling many intervals.

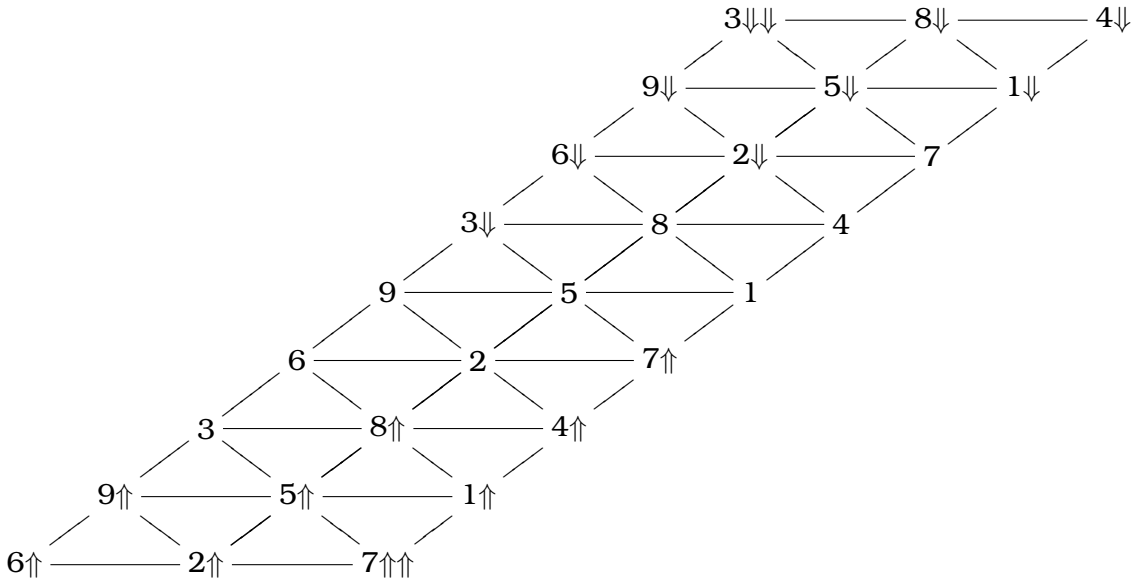
For example, here are some major triads.



The first two have the same pattern of scale steps if you ignore the river. This is the

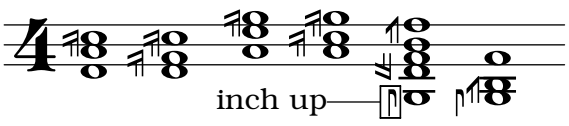
[2]This can be called a *wart* of the notation: an ugly feature that can't easily be removed.

Figure 3.5: A section of the lattice with simple, marvel spellings



better pattern for a major triad because it can be written using the nominals. Following it gives you the pitches shown in Figure 3.5. That isn't a very useful slice of the lattice. Sometimes you want to spell a chord differently, like the other two chords in the example. The final chord is the real problem: it can't be re-written correctly because lowering the top note by a scale degree will leave it on the wrong foot.

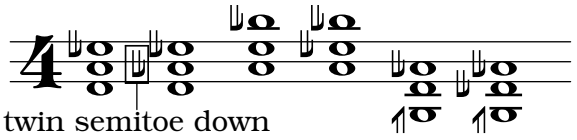
One way to get more correct chords is to use inch shifts. Pitches an inch apart are on the same scale degree but different feet. You can then use the much fatter lattice in Figure 3.6 and have a bit of freedom to move by fifths. Some problematic major and minor triads (and a full nine-limit otonality) can then be written as follows (accidentals don't persist).



This uses a different symbol for the inch shifts to Section 3.5. The symbol used there was for commas of Didymus (the difference between a major and minor tone). It isn't consistent with the spelling rules here so I chose a different Sagittal symbol that is.

A Red, Red Rose requires some inch spellings for general didymic harmony, and you can see it in Figure 3.6.

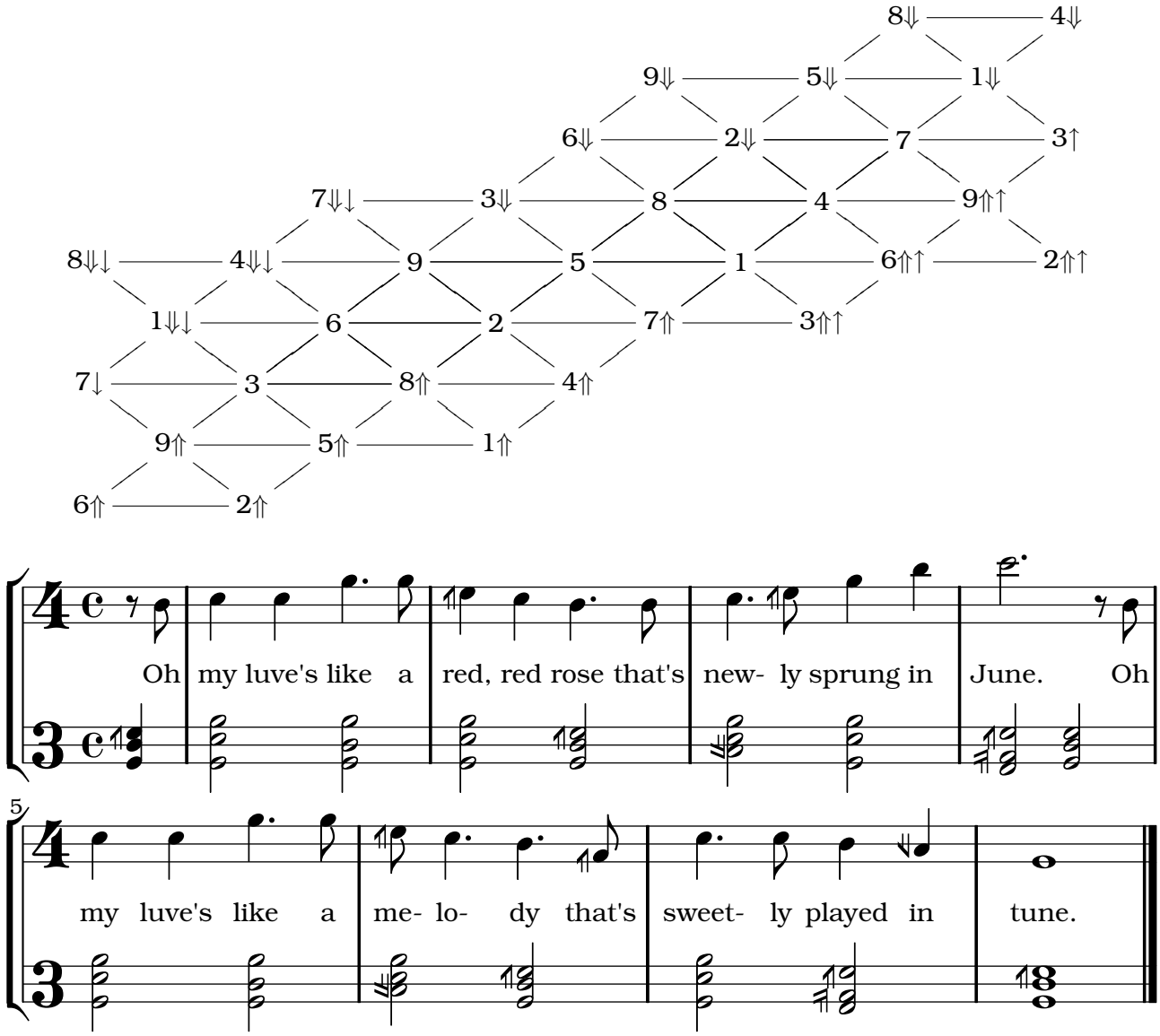
Another way to get more notes with marvel spelling is to use two different symbols for the different sized semitones. Some problem chords are then written like this:



The symbol for alternative semitoe shifts is the inverse of the one for standard shifts and curved. Because it turns a major third into a minor third, the downward shift has a common function with flats. For this reason I use sharp and flat signs on the lattice in Figure 3.7. This semitone/semitoe equivalence is close but remember that semitones with popular tunings are generally larger than semitoes so it's best not to use sharp and flat symbols in tripod notation.

Figure 3.7 also uses inch shifts, defined as Didymic commas. To make them distinct from the inches in Figure 3.6 I used "harpoons" that match the Sagittal symbols. This allows us to write a large number of pitches, with a fair amount of complexity. You can add whatever symbols you like from Athenian Sagittal but you could always use the same

Figure 3.6: A section of the lattice with marvel spellings using inch shifts and *A Red, Red Rose* using them

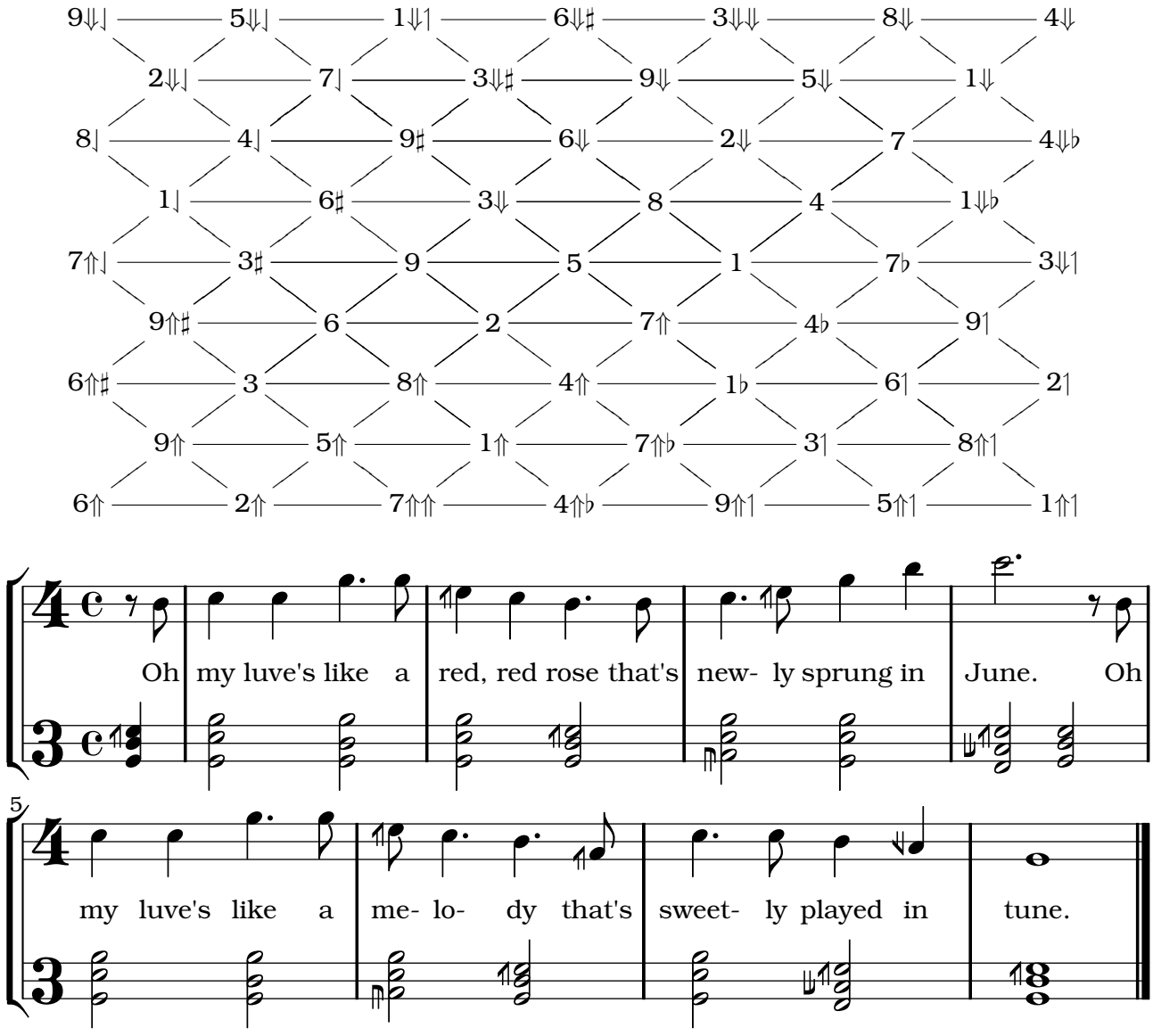


symbols on a conventional staff and it would be easier to read.

Some music still ends up simpler in tripod notation with the twin spellings of semitoes. The semitoe shifts can both be written with two shafts so if you miss the distinction you still have notation for magic temperament. Two semitoes in the same direction can always be three shaft symbols. Inches combining with semitoes give twin-headed two shaft symbols which makes them look larger than semitoes but smaller than toes, which is correct. There are two different symbols for inch shifts but none of the contexts I've outlined here require you to use both of them, and they're both single shaft symbols that look like hooks. I don't know what to do about semitoe shifts that combine in opposite directions—probably best to ignore them. Any nine-limit chord can be written correctly with only one set of magic nominals. If notes change their spelling from one chord to the next you can find a way to indicate this.

Fortunately, *A Red, Red Rose* doesn't require any of this complexity because the harmony is so simple. You can see it in Figure 3.7.

Figure 3.7: A section of the lattice with twin marvel spellings and *A Red, Red Rose* using them



4 Yan Tan Tethera Note Names

> I can't sing like that, not I! I can sing genuine music, like F and G; but not anything so much out of the order of nater as that.
>
> *Sammy Blore, Two on a Tower II (Thomas Hardy)*

4.1 Counting Sheep

THERE ARE MANY systems of special numbers associated with counting sheep in and around the north of England. The story goes that, after the Romans left, the Anglo-Saxon invaders settled mainly in the valleys of England and left hill farming to the native Britons. For centuries there were speakers of languages similar to Welsh living in England and not troubling the historical record. A tradition of using variations of British numbers to count sheep outlived these languages, and even after it stopped being used for practical purposes it survived as a way of amusing children.

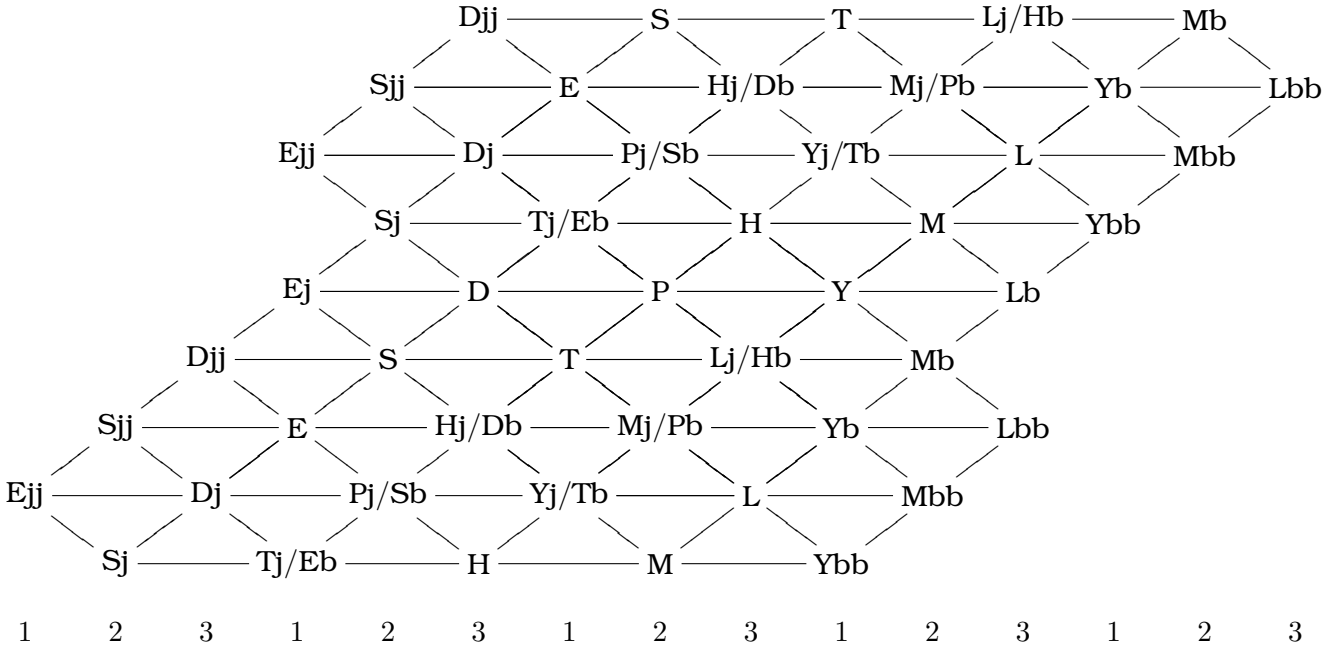
Whether this story is true or not doesn't concern us here. As these *Yan Tan Tethera* numbers have a certain musical quality to them I'll use them to name the tripod nominals. The results are in the table on the right. They're taken, for various reasons it would be too tedious to go into, from the "Derbyshire Dales" list given by *Wikipedia*.[1] You then raise a note by a semitoe by adding a "jig" to the name and lower it by adding a "bum". These suffixes follow from the words "bumfit" and "jiggit" used for the numbers fifteen and twenty in some variants. Double shifts become "bubum" and "jijig".

I think these names are more suitable than English letter names or numbers because they can't be confused with the traditional note names, or any of the ways numbers are used in music (including, of course, the foot annotations in this exposition).

	Full Name	Short	Terse	Trad.
1⇓⇓	Yanna-bubum	Yabub	Ybb	
1⇓	Yannabum	Yab	Yb	
1	Yan	Yan	Y	B
1⇑	Yannajig	Yaj	Yj	
2⇓	Tannabum	Tab	Tb	
2	Tan	Tan	T	C
2⇑	Tannajig	Taj	Tj	
3⇓	Tethabum	Eb	Eb	
3	Tethera	Eth	E	D♭
3⇑	Tethajig	Ej	Ej	D
3⇑⇑	Tetha-jijig	Ejij	Ejj	
4⇓⇓	Metha-bubum	Mebub	Mbb	
4⇓	Methabum	Meb	Mb	D
4	Methera	Meth	M	D♯
4⇑	Methajig	Mej	Mj	
5⇓	Pippabum	Pub	Pb	
5	Pip	Pip	P	E
5⇑	Pippajig	Pij	Pj	
6⇓	Sethabum	Seb	Sb	
6	Sethera	Seth	S	F
6⇑	Sethajig	Sej	Sj	
6⇑⇑	Setha-jijig	Sejij	Sjj	
7⇓⇓	Letha-bubum	Lebub	Lbb	
7⇓	Lethabum	Leb	Lb	
7	Lethera	Leth	L	G♭
7⇑	Lethajig	Lej	Lj	G
8⇓	Hovabum	Hob	Hb	G
8	Hovera	Hov	H	G♯
8⇑	Hovajig	Hoj	Hj	
9⇓	Dovabum	Dob	Db	
9	Dovera	Dov	D	A
9⇑	Dovajig	Doj	Dj	
9⇑⇑	Dova-jijig	Dojij	Djj	

[1] http://en.wikipedia.org/wiki/Yan_Tan_Tethera

Figure 4.1: A section of the lattice with Yan Tan Tethera names



4.2 Shorter Names

THE FULL Yan Tan Tethera names have a certain rhythm to them, but there are some contexts—like solfège—where they're too cumbersome. For this reason I added the shorter names in the "Short" column. Each note requiring a single shift is one syllable, making it easier to sing and type. Notes requiring a full toe shift are two syllables. Pippabum is shortened to "pub" as a special case because "pip" and "pib" can easily be confused.

Sometimes even the short names aren't short enough. For that reason I added the "Terse" column. Each nominal and each shift are written as a single letter to match traditional names (the last column). I renamed "teth" to "eth" so that each short name of a nominal starts with a different letter, making these terse names unique.[2] There is a precedent for this: some systems have "eddero" instead of "tethera". Maybe you'll prefer to use "ethera" for the full name as well.

One use for the terse names is to notate lattice points. For an example see Figure 4.1.

[2]You may still prefer to use "teb" for tethabum so that it doesn't look like E flat.

5 Coda

> In 256 BC . . . Prince Zhao of Qin State came to fetch the tripods. In fact, he had only seized eight and the ninth suddenly flew into the Sishui River . . . when the tripod was almost pulled out of the water, the celestial dragon in Sishui river wouldn't give it up and bit off the rope so that the tripod sank into the water again and was never seen after that.
>
> *Travel Round Jiangsu*

5.1 Sources

TRAVEL ROUND JIANGSU was published by the China Forestry Publishing House in 2001. The writers from Xuzhou are listed as Liu Shaogun*[sic]* (劉韶云), Zhao Hong (趙虹), Zhang Yan (張灩) and Shang Xuguang (單旭光).

The image on the front page is a processed version of a digital copy supplied by Wang Hongzhen (王洪震). It shows a rubbing of a stone relief, probably Han Dynasty, of the story told by the *Travel Round Jiangsu* quotes. Problems caused by the processing are, of course, my fault.

5.2 Implementation

I WROTE this exposition using Lilypondbook with LaTeX. You can download the full source code from `http://x31eq.com/magic/tripod-code.zip` which includes standalone LilyPond files, so you don't need to get LaTeX working. Also MIDI files using pitch bends!

Early versions (before February 6) used different Sagittal symbols. I'm afraid they really were wrong, so I made the change, which involves one symbol changing its meaning. This only affects Sections 3.5 and 3.6.

From October 21 the Sagittal URL is updated and I fixed a minor error.

This version was compiled on October 21, 2009.

5.3 Acknowledgements

THANK YOU to Hudson Lacerda for helping me get tripod notation working with MicroABC. I didn't end up using it here but never mind that. Some details were influenced by his comments.

He also mentioned something that Arnold Schoenberg called a *trigram* in *Style and Idea*. There are three spaces between each line, and you can write notes in the middle one using a ledger line. That gives you twelve notes to the octave without a need for accidentals. I didn't know about it before I thought of tripod notation, so it didn't influence me, but it does look very similar. I should mention it if only to stop others doing so.

My gratitude also to the Lilypond team for a program that handles tripod notation unreasonably well for software aimed at Common Practice notation. And Dave and George for the Sagittal symbols and the microtonal community in general for providing the background for these ideas to come out of.

5.4 Glossary

THERE ARE some terms in this exposition that are either uncommon or I invented. Some might be in the Tonalsoft Encyclopedia.[1] Here, anyway, are some brief definitions.

comma A small interval. In particular, the *comma of Didymus* (also known as the *syntonic comma*) that separates the two whole tones used in nine-limit harmony.

comma pump A chord sequence that leaves you a comma away from where you started.

Didymic Relating to conventional triadic harmony, and pitches related according to it. It's defined by an octave, perfect fifth and major third.

five-limit Equivalent to *Didymic*.

foot One of a threefold division of the octave. Each note of a triad stands on a different foot.

inch A small interval in magic temperament. Around forty to the octave, depending on the tuning.

lattice A regular pattern showing relationships between pitches.

magic A marvel temperament class with additional approximations.

marvel A nine-limit temperament class with the same structure as Didymic harmony.

meantone A way of tuning that gives each note in conventional notation a unique pitch with certain regularity. A marvel temperament class.

nine-limit harmony An extension of triadic harmony including some new intervals.

nominal A pitch you write without an accidental or key signature.

otonality A standard chord used to define a system of harmony.

Sagittal A comprehensive system of accidentals for microtonal music.

semitoe Half a toe. The interval between a major and minor third.

subminor A semitoe smaller than minor.

supramajor A semitoe larger than major.

river An extra ledger line or space or semitoe that separates the staves or registers of tripod notation.

temperament A way of tuning that approximates the ideal intervals of some harmonic system. In return for the approximation, you expect the tempered system to be simpler than the ideal one.

temperament class A set of temperaments that have the same structure but different tunings.

toe A standard interval of magic temperament. Equivalent to the semitone of a major scale, but on the large side. Around ten to the octave.

tripod A crazy system of notation with three feet.

Tonnetz The German word for *lattice* that's sometimes used in English.

tricycle A crazy system of notation using a traditional staff with foot indications.

wheel A foot in tricycle notation.

[1] http://tonalsoft.com/enc/encyclopedia.aspx